

Nuclear and Particle Physics - Problem Set 3 - Solution

Problem 1)

By setting $E' = \frac{E}{1 + \frac{2E}{M}(\sin \theta/2)^2} = 0.683 \text{ GeV}$, I find $Q^2 = 3.413 (\text{GeV}/c)^2$ and hence $\tau = \frac{Q^2}{4M^2} = 0.968$.

Using Eq. 5.39 in the book (replacing $|\mathbf{q}\mathbf{c}|^4$ with Q^4) and multiplying with the recoil factor E'/E as indicated in Eqs. 6.1-6.2 in the book, I get a Mott cross section of $4.53 \cdot 10^{-8} \text{ fm}^2 = 4.53 \cdot 10^{-34} \text{ cm}^2 = 453 \text{ pb}$ (picobarn).

From Eq. 6.12 in the book, we have

$$G_E^p = G_{\text{Dipole}} = 0.0297 \text{ and } G_M^p = 2.79 G_{\text{Dipole}} = 0.0828.$$

Plugging it all into Eq. 6.10 yields

$$d\sigma/d\Omega = 7.74 \cdot 10^{-36} \text{ cm}^2 = 7.74 \text{ pb (pico-barn). Very tiny!}$$

Problem 2)

Using the “point-like target” approximation, we calculate $d\Omega =$

$$(0.01\text{m})^2/(0.2\text{m})^2 = 0.0025 \text{ sr, hence } d\sigma = d\sigma/d\Omega d\Omega = 1.94 \cdot 10^{-38} \text{ cm}^2.$$

For the described target, we have $L = 5 \text{ cm}$, $\rho = 0.07085 \text{ g/cm}^3$ [Wikipedia], atomic weight = 1.008u [Wikipedia] and $N_A = 6.022 \cdot 10^{23} \Rightarrow$ areal density is $2.116 \cdot 10^{23} \text{ protons/cm}^2$. The beam flux is $100 \mu\text{A}/e = 6.242 \cdot 10^{14} \text{ electrons/s}$.

Hence the luminosity is $1.321 \cdot 10^{38}/\text{cm}^2/\text{s}$. Multiplying this with the cross section above, I get a count rate of 2.56/s. To achieve a precision of 10%, I

need to get $\frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} = 0.1$, so $N > 100$. This requires only 39 seconds of data taking, which is truly amazing given how small the cross section is!